Modeling of Surge Characteristics in Turbo Heat Pumps

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1 Introduction

 Turbo heat pumps are essentially large-scale cooling systems for buildings and use refrigerants as the working fluid. The system consists of a centrifugal compressor, expansion valve, and two heat exchangers: an evaporator and a condenser (Fig. 1(a)). The refrigerant absorbs heat from a low temperature reservoir (building interior) in the evaporator and releases heat to a high temperature reservoir (building exterior) in the condenser. Such systems are widely used for air-conditioning purposes throughout the world, especially in hot regions, and, like other compression systems, they can suffer from surge.

 Surge is a dynamic system instability that can occur at off-design conditions in various compression and pumping systems, including axial and centrifugal compressors, pumps, and turbopumps. In turbo heat pumps, the temperature of the building exterior (i.e., atmosphere) determines the temperature and corresponding saturation pressure in the condenser. Therefore, if the atmospheric temperature rises abnormally above that for which the heat pump was designed, the condenser pressure becomes higher than the design condenser pressure. Thus, the compressor exit pressure rises, and the compressor can encounter surge. Surge, especially deep surge involving flow reversal, can be noisy and cause mechanical failure of components. In gas turbine engines, the hot gas from the combustor can destroy the compressor.

 In turbo heat pumps, liquid refrigerant flow reversal during surge has been known to severely damage the impeller and/or bearings. A gas turbine is an open-loop system with one plenum (combustor) and an ideal gas (single-phase) as the working fluid. Compared with such gas turbines, turbo heat pumps introduce additional complexities for the following reasons. First, the turbo heat pump forms a closed-loop system. Thus, the compressor inlet condition is affected by the compressor outlet condition and, therefore, not constant. Second, the system has a condenser and an evaporator that are coupled to each other and can act as plenums for refrigerant storage. Third, the heat pump runs on a vapor-compression refrigeration cycle with two phases: vapor and liquid. In the evaporator and condenser, vapor and liquid phases are separated by a free surface, and the volume of each phase can change. Also, a two-phase flow exists at the evaporator inlet. Fourth, phase change heat transfer occurs in the evaporator and condenser. Thus, not only mechanical energy input (from the compressor) but also heat transfer effects (to and from the surroundings) need to be considered. Fifth, unlike air, a refrigerant has strong real gas effects and thus cannot be modeled as an ideal gas. Properties, such as density, speed of sound, and enthalpy, have to be calculated with change of pressure.

 Much research has been conducted on surge and rotating stall in gas turbines. Greitzer [1,2] analytically and experimentally investigated surge and rotating stall in compression systems. Surge involves large amplitude fluctuations of annulus averaged pressure and mass flow rate in time. On the other hand, rotating stall has constant annulus averaged properties but circumferentially fluctuating flow. Greitzer’s analytical model consists of an axial compressor, plenum, throttle, and ducting. The working fluid is air, and the system inlet and outlet are open to atmosphere. The model is capable of accurately predicting surge and rotating stall in gas turbines. Hansen et al. [3] showed that the Greitzer model is also applicable to describing surge in systems with centrifugal compressors. Gysling et al. [4] stabilized surge in a centrifugal compressor using a variable volume plenum. Botha et al. [5] developed a surge model for both open-loop and closed-loop Brayton cycle systems by coupling the compressor and turbine. They took into account heat exchange as well as momentum effects. In all of the studies above, the assumed working fluid was air, an ideal gas.

 Instabilities in pumping systems with liquids as working fluid have also been examined. Rothe and Runstadler [6] developed theoretical models to explain surge in pumps. Although the fluid passing through the pump and throttle is liquid, the compressibility from the gas (air) in the compressor outlet tank yields pressure and pump mass flow rate fluctuations. They also conducted experiments and validated their model. Instabilities in pumping systems with two-phase flows have also been examined. For example, Tsujimoto et al. [7] examined how cavitation affects instabilities in turbopumps.

 Instabilities during phase change have also been investigated. Kakaç and Bon [8] developed analytical models of two-phase flow instabilities in tube boiling systems. The system consisted of
a surge tank followed by a heater section and exit to atmosphere. They explained several types of instabilities with intersections and slopes of the characteristic curves of the tube and pump. 

Transient responses of heat pumps have been investigated but mostly from the heat transfer point of view. Chi and Didion [9] studied transient performance of air-to-air heat pumps. They assumed a quasi-steady compressor performance and simulated the start-up transients of the heat pump. Predictions matched the experimental data well. Bendapudi et al. [10] investigated transients in a turbo heat pump. The compressor was assumed to operate at steady-state condition, and the transient responses of heat exchangers were investigated.

Despite such efforts, there has been little previous work on surge in turbo heat pumps. Therefore, this paper introduces a new analysis of surge in turbo heat pumps. The main objectives are (1) to develop an analytical model based on the first principles that is capable of predicting and (2) to identify nondimensional parameters that influence surge behavior in turbo heat pumps. In the following sections, the new model is described, and the model’s predictions are validated through comparison with experimental data. Finally, the influence of Greitzer’s B parameter and the ratio of heat exchanger volumes are examined.

2 Model Description

Figure 1(b) shows the cycle diagram of a typical turbo heat pump system. From the compressor exit, station a, the compressed refrigerant vapor flows into the condenser and releases heat to the cooling water and changes to liquid phase. The liquefied refrigerant is then vaporized upon absorbing heat from the chilled water in the evaporator before returning to the compressor.

The assumptions made in the model are as follows.

1. The flow is inviscid and one-dimensional.
2. The refrigerant is R134a.
3. For each phase, the flow is spatially uniform.
4. Pressure drop across condenser, evaporator, and ducting are negligible.
5. Gravity effects are negligible.
6. Mass flow rates and inlet temperatures of cooling (condenser) and chilled (evaporator) waters are constant.
7. The condenser and evaporator are assumed to be shell-and-tube type heat exchangers with the shell containing the refrigerant and tubes passing the cooling and chilled waters, respectively (Fig. 2).
8. At the condenser inlet, the cooling water temperature is lower than the saturation temperature of the refrigerant in the condenser. At the condenser exit, the cooling water’s temperature is equivalent to the refrigerant’s saturation temperature.
9. At the evaporator inlet, the chilled water temperature is higher than the saturation temperature of the refrigerant in the evaporator. At the evaporator exit, the chilled water’s temperature is equivalent to the saturation temperature of the refrigerant.
10. At the exit of the expansion valve (c), a two-phase mixture is assumed. Refrigerant at condenser inlet and evaporator exit is assumed to be saturated vapor. At the condenser exit, saturated liquid is assumed. At the compressor exit, the refrigerant is actually slightly superheated (Fig. 1(b)); however, the amount of superheat is small and thus assumed to be negligible.

The modeling process is similar in philosophy to the model developed by Greitzer [1]. The momentum equations across the compressor and expansion valve are the same as Greitzer’s. One difference is that the compressor inlet pressure \( P_2 \) is not constant but variable in the new closed-loop model.

\[
-P_1 + P_2 + C = \frac{L_c}{A_c} \frac{dn_c}{dt} \tag{1}
\]

\[
P_1 - P_2 = F = \frac{L_t}{A_t} \frac{dm_t}{dt} \tag{2}
\]

Compressor pressure rise \( C \) is a function of mass flow rate \( m_c \), and it is obtained experimentally. Pressure drop \( F \) can be written as follows:

\[
F = \frac{m_t^2}{2p_{2t}A_s^2} \tag{3}
\]

The compressor inlet duct area is not uniform. By defining the effective length and area, the effective inertia can be obtained.
The compressor inlet area is chosen as $A_C$, and $L_C$ is calculated by Eq. (4). The same process is conducted for the expansion valve duct.

The new model has four continuity equations: one for each phase in the condenser and evaporator. Furthermore, there are two heat balance equations, one for each heat exchanger, between the cooling/chilled water and the refrigerant. The two continuity equations for the condenser are

$$m_C - m_{l1} = \frac{d}{dt}(p_{t1}V_{t1})$$

(5)

$$m_{l1} - m_T = \frac{d}{dt}(p_{l1}V_{l1})$$

(6)

where $m_{l1}$ is the rate of saturated liquid refrigerant generation via condensation. Not only density but also volume of each phase changes. The sum of the two volumes is equivalent to the condenser volume. Therefore, the volume change rates of the two regions are related to each other as

$$dV_{t1} = -dV_{l1}$$

(7)

Using the definition of speed of sound, the density change can be written in terms of pressure change as follows:

$$\dot{m}_C - \dot{m}_{l1} = \frac{V_{l1}dP_{l1}}{A_{l1}} + \frac{dV_{l1}}{dt}$$

(8)

$$\dot{m}_{l1} - \dot{m}_T = \frac{V_{l1}dP_{l1}}{A_{l1}} - \frac{dV_{l1}}{dt}$$

(9)

The liquid generation rate $\dot{m}_{l1}$ can be calculated from the energy balance between the refrigerant and cooling waters. The heat released from the refrigerant, or the latent heat, is absorbed by the cooling water.

$$\dot{m}_{l1}h_{fl} = \dot{m}_{cw}C_{fw}(T_1 - T_{cw,0})$$

(10)

The cooling water inlet temperature and mass flow rate are given, and the outlet temperature can be calculated as a function of pressure $P_1$.

In the condenser, the liquefied refrigerant does not contribute to liquid generation rate $\dot{m}_{l1}$. Instead, $\dot{m}_{l1}$ is a function of vapor volume in the condenser $V_{t1}$

$$\dot{m}_{l1}h_{fl} = g_1\dot{m}_{cw}C_{fw}(T_1 - T_{cw,0})$$

(11)

where $g_1 = g_1(V_{t1})$ is proportional to $V_{t1}$, and $V_{t1}$ can be obtained from Eqs. (8) and (9).

Similarly, in the evaporator

$$\dot{m}_{l2} - \dot{m}_C = \frac{V_{l2}dP_{l2}}{A_{l2}} + \frac{dV_{l2}}{dt}$$

(12)

$$\dot{m}_T - \dot{m}_{l2} = \frac{V_{l2}dP_{l2}}{A_{l2}} - \frac{dV_{l2}}{dt}$$

(13)

The energy balance between the refrigerant and chilled waters in the evaporator is

$$\dot{m}_{l2}h_{fl,2} = \dot{m}_{cw}C_{fw}(T_{cw,0} - T_2)$$

(14)

In the evaporator, the vapor region has no effect on $\dot{m}_{l2}$. Only the volume of the liquid refrigerant in the evaporator $V_{l2}$ contributes to phase change.

The chilled water temperature at the evaporator outlet $T_2$ is the same as the saturation temperature of the refrigerant in the evaporator and thus is a function of saturation pressure $P_2$. The liquid refrigerant in the evaporator is in two-phase state. The two-phase properties can be expressed in terms of the refrigerant quality $x$, and are denoted with subscript $x$ (Appendix). Quality is calculated as a function of the expansion valve inlet and outlet pressures $P_1$ and $P_2$, assuming that enthalpy remains constant across the expansion valve.

The last equation is related to the transient compressor response. In the region where the compressor characteristic has a positive slope, the compressor pressure rise $C$ cannot instantaneously respond to the changes in $\dot{m}_C$. According to Greitzer [1], this phenomenon can be modeled as follows:

$$\frac{dC}{dt} = C_{ss} - C$$

(15)

where $C_{ss}$ is the steady-state compressor pressure rise, and $\tau$ denotes the time-lag.

The system of nine equations (Eqs. (1), (2), (8), (9), and (11)–(15)) can be nondimensionalized as follows. The nondimensionalized variables are

$$\bar{m} = \frac{m}{\rho_{lv0}UA_{C}}, \quad \bar{p} = \frac{P}{1/2\rho_{lv0}U^2}, \quad \bar{T} = \omega t$$

$$\bar{V}_{1e(1)} = \frac{V_{1e(1)}}{V_{1e,0}}, \quad \bar{V}_{2e(2)} = \frac{V_{2e(2)}}{V_{2e,0}}$$

$$\bar{\rho} = \frac{\rho}{\rho_{lv0}}, \quad \bar{h}_{fl} = \frac{h_{fl}}{U^2}$$

$$\bar{a}_{1e(1)} = \frac{a_{1e(1)}}{a_{1e,0}}, \quad \bar{a}_{2e(2),a} = \frac{a_{2e(2),a}}{a_{2e,0}}$$

$$\bar{T}_1 = \frac{T_1}{T_{cw,0}}, \quad \bar{T}_2 = \frac{T_2}{T_{ch,0}}$$

The corresponding nondimensional equations are

$$\frac{d\bar{m}_C}{dt} = B\left[-\bar{\dot{P}}_1 + \bar{\dot{P}}_2 + \bar{\dot{C}}\right]$$

(16)

$$\frac{d\bar{m}_T}{dt} = \frac{G}{\bar{B}} \left[ \bar{\dot{P}}_1 - \bar{\dot{P}}_2 - K \frac{\bar{m}_T^2}{\bar{a}_{2e,0}} \right]$$

(17)

$$\frac{d\bar{\dot{P}}_1}{dt} = \frac{1}{\bar{B}} \bar{a}_{1e,1} \left[ \bar{m}_C - \left( \frac{\bar{\rho}_{lv}}{\bar{\rho}_{l1}} - 1 \right) \bar{m}_{l1} \right]$$

(18)

$$\frac{d\bar{\dot{P}}_2}{dt} = \frac{G}{\bar{B}} \left( \frac{\omega}{\omega_1} \right) \frac{\bar{a}_{2e,0}^2}{\bar{a}_{2e,1}^2} \left( \bar{\dot{V}}_{l1} + \bar{\dot{V}}_{l2} - m_{l1} \right) - \bar{\dot{V}}_{l1} - \bar{\dot{V}}_{l2} - \bar{\dot{V}}_{l1}$$

(19)

$$\frac{d\bar{\dot{V}}_{l1}}{dt} = \frac{1}{2} \frac{\bar{a}_{2e,1}^2}{\bar{a}_{1e,1}^2} \left( \bar{\dot{V}}_{l1} - \bar{\dot{V}}_{l2} - \bar{\dot{V}}_{l1} \right) - \bar{\dot{V}}_{l1}$$

(20)

$$\frac{d\bar{\dot{V}}_{l2}}{dt} = \frac{G}{\bar{B}} \left( \frac{\omega}{\omega_1} \right) \frac{\bar{a}_{2e,0}^2}{\bar{a}_{2e,1}^2} \left( \bar{\dot{V}}_{l1} + \bar{\dot{V}}_{l2} - \bar{\dot{V}}_{l1} \right) - \bar{\dot{V}}_{l1} - \bar{\dot{V}}_{l2} - \bar{\dot{V}}_{l1}$$

(21)

$$\bar{m}_{l1} = \bar{g}_{1} \bar{H}_{1} \left( \frac{T_1 - 1}{\bar{\rho}_{fl}} \right)$$

(22)
\[ \ddot{m}_{c2} = g_2 H_2 \frac{(1 - \bar{T}_2)}{\bar{h}_{fg,2}} \]  
(23)

\[ \frac{d\bar{C}}{dt} = \bar{C}_{SS} - \bar{C} \]  
(24)

where

\[ \frac{\bar{V}_1}{\bar{a}_1} = \frac{\bar{V}_{1w}}{\bar{a}_{1w}} + \frac{\bar{p}_{1u}}{\bar{p}_{1i}} \frac{\bar{V}_1}{\bar{a}_{1i}} \]  
(25)

\[ \frac{\bar{V}_2}{\bar{a}_2} = \frac{\bar{V}_{2u}}{\bar{a}_{2u}} + \frac{\bar{p}_{2u}}{\bar{p}_{2i}} \frac{\bar{V}_2}{\bar{a}_{2i}} \]  
(26)

Furthermore, the nondimensional input parameters that govern system stability are

\[ B = \frac{U}{2a_{1,0} L_C}, \quad G = \frac{L_f \lambda_C}{L_C \lambda_T} \]

\[ K = \frac{A_C}{A_1}, \quad \tau = \frac{\pi NR}{L_C B} \]

\[ M_{a1} = \frac{U}{a_{1,0}}, \quad M_{a2} = \frac{U}{a_{2,0}} \]

\[ H_1 = \frac{\bar{m}_{cw,T_cw}}{U^2}, \quad H_2 = \frac{\bar{m}_{chw, T_{chw}}}{U^2} \]

\[ \frac{h_{2,u}}{h_{2,i}} = a_{2,0} \sqrt{a_{1,0}^2} \frac{A_f}{V_{2,0} L_f} \left( \frac{A_C}{V_{1,0} L_C} \right) \]

The \( B, G, K, \) and \( \tau \) parameters appear in Greitzer’s model as well. The newly found parameters for turbo heat pumps are \( M_{a1}, M_{a2}, H_1, H_2, \) and \( \omega_{2}/\omega_1. \)

### 3 Model Predictions

This section presents predictions from the new model. First, the model is applied to a single-phase open-loop system. Both linear and nonlinear analysis results are compared with predictions from the Greitzer model. Second, the new model is applied to a modern turbo heat pump system, and the model’s predictions are validated through comparison with experimental data. Finally, a parametric study is conducted to examine the effects of Greitzer’s \( B \) parameter and relative volumes of condenser and evaporator \( \omega_2/\omega_1. \)

#### 3.1 Comparison With Greitzer Model

Equations (16)–(24) include nonlinear equations. The nonlinearity arises mainly from the compressor characteristic curve and changes in the volume, heat exchange, and other properties; and yet, much insight about this physical phenomenon can be obtained from a linearized analysis. Therefore, a linearized analysis is conducted to identify key parameters.

All of the variables are assumed to consist of mean (\( \bar{()}) \) and perturbation (\( \delta() \)) components. Compressor and expansion valve characteristics are expressed as first-order functions of mass flow rate.

\[ \tilde{C} = \bar{C} + (d\bar{C}/d\bar{m}) \delta \tilde{m}_c \]

\[ \tilde{F} = \bar{F} + (d\bar{F}/d\bar{m}) \delta \tilde{P}_f \]

Also, the phase change mass flow rates are expressed as functions of saturation pressure.

\[ \tilde{m}_{m1} = \bar{m}_{m1} + (d\bar{m}_{m1}/d\bar{P}) \delta \bar{P}_1 \]

\[ \tilde{m}_{m2} = \bar{m}_{m2} + (d\bar{m}_{m2}/d\bar{P}) \delta \bar{P}_2 \]

Eliminating the mean values, the system of equations in Eqs. (16)–(24) can be reduced to Eq. (31).

\[ \begin{bmatrix} d\delta \bar{m}_C \\ dt \\ \hline d\delta \bar{P}_1 \\ dt \\ \hline d\delta \bar{F} \\ dt \\ \hline d\delta \bar{P}_2 \\ dt \\ \hline \end{bmatrix} = \begin{bmatrix} B (d\bar{C}/d\bar{m}) \\ \hline 1 \left( \frac{B}{\bar{a}_1 V_1} \right) \\ \hline 1 \left( \frac{B}{\bar{a}_2 V_2} \right) \\ \hline 0 \\ \hline 0 \left( \frac{G}{\bar{a}_1} \right) \left( \frac{\bar{a}_2 V_2}{V_2} \right) \\ \hline 0 \left( \frac{G}{\bar{a}_2} \right) \left( \frac{\bar{a}_1 V_1}{V_1} \right) \end{bmatrix} \begin{bmatrix} \delta \bar{m}_C \\ \delta \bar{P}_1 \\ \delta \bar{F} \\ \delta \bar{P}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ \hline -B \frac{1}{\bar{a}_1} (d\bar{C}/d\bar{m}) \tilde{m}_{m1} \delta \bar{P}_1 \delta \bar{P}_2 \\ \hline -B \frac{1}{\bar{a}_2} (d\bar{F}/d\bar{m}) \tilde{m}_{m2} \delta \bar{P}_1 \delta \bar{P}_2 \\ \hline 0 \\ \hline -B \frac{1}{\bar{a}_1} (d\bar{C}/d\bar{m}) \tilde{m}_{m1} \delta \bar{P}_1 \delta \bar{P}_2 \end{bmatrix} \begin{bmatrix} 0 \\ \bar{F} \frac{B}{\bar{a}_1} \left( \frac{\bar{a}_2 V_2}{V_2} \right) \tilde{m}_{m1} \delta \bar{P}_1 \delta \bar{P}_2 \\ \bar{F} \frac{B}{\bar{a}_2} \left( \frac{\bar{a}_1 V_1}{V_1} \right) \tilde{m}_{m2} \delta \bar{P}_1 \delta \bar{P}_2 \\ 0 \\ \bar{F} \frac{B}{\bar{a}_1} \left( \frac{\bar{a}_2 V_2}{V_2} \right) \tilde{m}_{m1} \delta \bar{P}_1 \delta \bar{P}_2 \end{bmatrix} \begin{bmatrix} \delta \bar{m}_C \\ \delta \bar{P}_1 \\ \delta \bar{F} \\ \delta \bar{P}_2 \end{bmatrix} \]

If we impose the following conditions to the turbo heat pump system model in Eq. (31):

1. no heat transfer: \( \tilde{m}_{m1} = \tilde{m}_{m2} = 0 \)
2. fixed compressor inlet pressure: \( \delta \bar{P}_1 = 0 \)
3. single-phase condition: \( \bar{V}_{1i} = \bar{V}_{2i} = 0 \)
4. negligible inertia in the expansion duct: \( d\delta \bar{m}_C/ dt = 0 \)

Equation (31) can be reduced to Eq. (32), which is identical to the Greitzer model for a single-phase open-loop system.
Thus, the new model shows potential for applicability to a diverse set of systems: open-loop single-phase system, open-loop two-phase system, closed-loop single-phase system, closed-loop two-phase system, systems with one or two plenums, systems with and without heat exchange, etc.

Looking at Eq. (31), a turbo heat pump system can be thought of as two coupled subsystems: \((\delta \tilde{\eta}_C, \delta \tilde{P}_1)\) and \((\delta \tilde{\eta}_T, \delta \tilde{P}_2)\). One is the compressor-condenser system, and the other is the expansion valve-evaporator system. The two systems are coupled because \(\delta \tilde{\eta}_C\) and \(\delta \tilde{P}_1\) are functions of \(\delta \tilde{\eta}_T\) and \(\delta \tilde{P}_2\), and vice versa. Also, the parameters that influence stability can be identified from the linear analysis. In addition to the \(B\) parameter, geometric parameters \(G\) and \(K\) (implicit in \((d \tilde{F}/d \tilde{m})\)), plenum volume ratio \(\omega_2/\omega_1\), and heat transfer \(H\) (implicit in \((d \tilde{m}_C/d \tilde{P})\)) determine stability.

The new model has been applied to acquire instability predictions for a simplified single-phase, open-loop system. The system’s compressor characteristic and input parameters are given in Ref. [11] and are listed in Table 1.

The same system has been analyzed with two values of \(B = 0.6\) and 1.58. Figure 3(a) shows predictions from the Greitzer model and the new model for \(B = 0.6\). Figure 3(b) shows the corresponding predictions for \(B = 1.58\). As expected, both models predict rotating stall for \(B = 0.6\) and deep surge for \(B = 1.58\). Furthermore, the new model’s predictions agree well (qualitatively and quantitatively) with those from the Greitzer model. Thus, the new model accurately captures the effect of \(B\), or the transition between rotating stall and surge. In summary, the new model can accurately predict instability behavior in the limiting case of an open-loop single-phase compression system.

### Table 1 Nondimensional Parameter Values

| \(B\) | \(B = 0.6, 1.58\) |
| \(G\) | 0.36 |
| \(K\) | 5.7 |
| \(\dot{\tau}\) | 1.8599, 0.706 |

### Table 2 Nondimensional Parameter Values

| \(B\) | 0.639 |
| \(G\) | 6.34 |
| \(K\) | 13,000 |
| \(\dot{\tau}\) | 0.0077 |
| \(\omega_2/\omega_1\) | 0.227 |
| \(\text{Ma}_a\) | 0.82 |
| \(\text{Ma}_b\) | 0.78 |
| \(H_1\) | 79.49 |
| \(H_2\) | 56.97 |

#### 3.2 Validation Versus Experimental Data

The model has been applied to a modern turbo heat pump system. The system of equations (Eqs. (16)–(24)) has been solved via a fourth order Runge–Kutta method. Input data for the model, including the compressor performance curve, have been obtained from LS Mtron Co., Ltd. (Gyeonggi-do, Korea) [12]. The nondimensional parameter values used in the analysis are listed in Table 2. The time-varying properties, such as density, speed of sound, quality, and saturation temperature, are calculated as functions of pressure and are updated using REFPROP, a commercial software program [13].

The LS Mtron test-rig consists of a centrifugal compressor with a vane diffuser, shell-and-tube type condenser and evaporator, and orifice plate as expansion device. The sensor locations are indicated in Fig. 1(a). In the figure, \(T\), \(P\), and \(F\) refer to temperature sensors, pressure sensors, and mass flowmeters, respectively. Siemens 7MF4332 pressure transmitters are installed at compressor inlet and outlet, and the top of the heat exchangers (to measure vapor pressures in the condenser and the evaporator). RTD sensors are used to measure temperatures at the compressor inlet and outlet, and cooling and chilled water inlets and outlets. The mass flow rates of cooling and chilled waters are measured using electromagnetic flowmeters. A Krohne OPTIFLUS 4300 C flowmeter is installed at the cooling water inlet, and a Siemens 7SM5030 flowmeter is installed at the chilled water inlet. The velocity of refrigerant at the compressor exit \(C_v\) is measured with an Oval VF vortex flowmeter. The vortex flowmeter provides the magnitude but not the direction of mass flow. Therefore, flow reversal was determined by comparing the compressor inlet and exit temperatures, as in Ref. [14].

Figure 4(a) shows graphs of nondimensional axial velocity at the compressor exit and pressure rise through the compressor plotted versus nondimensional time. For velocity and pressure rise, both the magnitude and the frequency of fluctuations are accurately predicted by the new model. Figure 4(b) shows the predicted and measured surge cycle along with the compressor characteristic curve. The predicted cycle has a deep surge shape. At the highest pressure rise point, the mass flow rate suddenly shifts to the maximum negative mass flow rate point. Then, the mass flow rate increases and the pressure rise decreases along the compressor characteristic for negative flow rates. At the lowest pressure rise point, the mass flow rate rapidly attains the maximum positive value. Subsequently, mass flow rate is decreased and the pressure rise is increased along the steady compressor characteristic, and the cycle is repeated. The experimental data are concentrated in two specific regions: the highest and lowest pressure rise regions. The fast changing regions, the lowest and highest mass flow rate regions, have not been well captured experimentally because of the limited frequency response of sensors and sampling frequency. Nevertheless, the overall trajectory of the surge cycle is well predicted by the model. In open-loop systems, the compressor inlet pressure remains constant. Thus, the change in \(\Delta P\) is caused by the outlet pressure change. However, in this closed-loop system, not only the compressor outlet pressure but also the inlet pressure changes, as seen in Fig. 4(c).

The chilled water temperature at the evaporator inlet is higher.
than that at the evaporator outlet during normal design point operation of the heat pump. However, during the flow reversal stage of surge, the chilled water temperature at the evaporator exit has been observed to become higher than that at the evaporator inlet.
The same trend is predicted by the new model. As the flow reversal occurs in the compressor, the compressor inlet pressure $P_2$ is increased (Fig. 4). The increased compressor inlet pressure increases the evaporator saturation pressure and saturation temperature. Since the chilled water temperature at the evaporator exit cannot be lower than the evaporator saturation temperature, the chilled water temperature at the evaporator exit increases and becomes higher than that at the evaporator inlet. The model also predicts that the higher exit chilled water temperature leads to a negative value of $\tilde{m}_s$, which means condensation instead of evaporation occurs in the evaporator during flow reversal. Consequently, instead of cooling, the turbo heat pump ends up heating the building during the flow reversal. This effect is manifested as negative $\dot{Q}_{L}$ value, and this trend is visible in Fig. 4.

### 3.3 Parametric Study

#### 3.3.1 $B$ Effect

The effect of $B$ parameter has been examined. Figure 5 shows the graphs of mass flow rate and pressure rise plotted versus time as well as the surge cycle for $B=0.0256$. Figure 6 shows the corresponding set of figures for $B=0.153$. In Figs. 5 and 6, the amplitudes of pressure rise fluctuations are similar. However, the amplitude of mass flow fluctuation for $B=0.0256$ is slightly smaller than that for $B=0.153$. Thus, the oval shape of the surge cycle for $B=0.0256$ becomes a parallelogram for $B=0.153$. According to Greitzer [1], $B$ can be thought of as the ratio of the pressure and inertial forces in the system

$$B = \frac{1}{2} \frac{p_{20,0} U^2 A_C}{\rho_{25,0} \omega \sqrt{UL_C A_C}}$$

and higher $B$ leads to larger axial velocity perturbations. Thus, flow reversal occurs for both values of $B$ for this turbo heat pump but the surge becomes deeper as $B$ increases. Furthermore, the frequency of surge decreases as $B$ increases. Such influence of $B$ in turbo heat pumps is similar to that found in gas turbines.

#### 3.3.2 $\omega_2/\omega_1$ Effect

The Helmholtz frequencies of two plenums (condenser and evaporator) also influence surge behavior. The $B$ parameter influences both the shape and the frequency of the surge cycle. Unlike the $B$ parameter, $\omega_2/\omega_1$ influences the frequency of the surge cycle more than the shape. In fact, the shape of the surge cycle is almost identical for different values of $\omega_2/\omega_1$. The frequency of surge cycle, however, is very sensitive to the change in $\omega_2/\omega_1$. Figure 7 shows the $\omega_2/\omega_1$ effect on the frequency of surge in log scale. The nondimensional frequency changes exponentially as $\omega_2/\omega_1$ changes. From the curve fitting of the data in Fig. 7, Eq. (34) can be obtained for this heat pump system.

$$\text{frequency} = \left[ \frac{1}{10} \left( \frac{\omega_2}{\omega_1} \right) \right]^{0.93}$$

If $G$ is constant, $\omega_2/\omega_1$ is proportional to the square root of the condenser volume normalized by the evaporator volume.
fluctuations and leads to a higher surge frequency. On the other hand, smaller volume has less capacity to bufferstore more refrigerant, and the frequency of surge is decreased.

\[ \frac{\omega_2}{\omega_1} = a_2 \sqrt{\frac{A_T}{V_{2,0} L_T}} a_{1,0} \sqrt{\frac{A_C}{V_{1,0} L_C}} = \frac{1}{\sqrt{G}} \sqrt{\frac{V_{1,0}}{V_{2,0}}} \]  

Inserting Eq. (35) to Eq. (34) gives

\[ \text{frequency} \approx \frac{V_{1,0}}{V_{2,0}} \]  

Thus, surge frequency is proportional to the ratio of the condenser volume normalized by the evaporator volume. The Helmholtz frequency parameter \( \omega_2/\omega_1 \) serves as the coupling factor between the two dynamic systems in the heat pump system (Eq. (31)). The \( B \) parameter is the ratio of external force and compressor outlet inertia. For a fixed value of \( B \), the compressor outlet characteristics including the condenser volume is fixed. Therefore, in such cases, the ratio of two volumes can only be changed by varying the volume of the evaporator \( V_{2,0} \). Larger volume can store more refrigerant, and the frequency of surge is decreased. On the other hand, smaller volume has less capacity to buffer fluctuations and leads to a higher surge frequency.

4 Conclusions

The new conclusions from this study are as follows.

1. A new analytical model, based on first principles, has been developed to predict instability in two-phase, closed-loop systems with multiple plenums and heat transfer (e.g., turbo heat pump).
2. The model has been validated against available experimental data, and the model can accurately predict surge in a turbo heat pump system.
3. During the flow reversal stage of surge, condensation can occur in the evaporator. Thus, the turbo heat pump can end up heating the building during this stage.
4. The well-known \( B \) parameter influences both surge shape and frequency.
5. As the ratio of the Helmholtz frequency of the evaporator to condenser (or the square root of the ratio of the volume of the condenser to evaporator) increases, the surge frequency increases exponentially.
6. The model can also accurately predict instability in an open-loop, single-phase system (e.g., gas turbine).
7. The new model can also be applied to a diverse set of systems (e.g., single-phase open-loop, single-phase closed-loop, systems with and without heat transfer, etc.).

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