Surge Onset in Turbo Heat Pumps

A typical turbo heat pump system consists of a centrifugal compressor, expansion valve, and two heat exchangers—a condenser and evaporator. Compared to a gas turbine, a turbo heat pump introduces additional complexities because it is a two-phase closed-loop system with heat exchange using a real gas/liquid (refrigerant) as the working fluid. For the first time, surge onset in such systems has been physically, analytically, and experimentally investigated. This study analytically investigates the physical mechanisms of surge onset in turbo heat pumps. From an existing nonlinear turbo heat pump surge model, the turbo heat pump is viewed as a mass-spring-damper system with two inertias, two dampers, and four springs which is then further simplified to a single degree-of-freedom system. Surge onset occurs when the system damping becomes zero and depends not only the compressor but also on the ducts, heat exchangers, and expansion valve. Alternatively, a new stability model has been developed by applying a linearized small perturbation method to the nonlinear turbo heat pump surge model. When the new linear stability model is applied to a conventional open loop compression system (e.g., a turbocharger), predictions identical to those of Greitzer’s model are obtained. In addition, surge onset has been experimentally measured in two turbo heat pumps. A comparison of the predictions and measurements shows that the mass-spring-damper model and the linearized stability model can accurately predict the turbo heat pump surge onset and the mass-spring-damper model can explain the turbo heat pump surge onset mechanisms and parametric trends in turbo heat pumps. [DOI: 10.1115/1.4026145]

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Introduction

Compression systems are used in various applications ranging from aircraft gas turbine engines to industrial compressors. Among these, turbo heat pumps have recently gained attention due to the rapid rise in the global demand for large-scale building cooling systems. Turbo heat pumps move heat from a low temperature reservoir (building interior) to a high temperature reservoir (building exterior). Refrigerants run through the system consisting of a centrifugal compressor, expansion valve, and two heat exchangers—an evaporator and condenser. Similar to other compression systems, turbo heat pump compressor operation is limited by surge. Surge is a one-dimensional system instability that depends not only on the local compressor fluid dynamics but also on the dynamic properties of the entire system: compressor, inlet and outlet duct, plenum, and throttle. During surge, a large-amplitude limit-cycle oscillation is typically encountered [1]. Thus, surge can cause a loss in performance along with fluctuating mechanical and thermal loads. In turbo heat pumps, surge can destroy the impeller and/or bearings. Therefore, compressors are typically operated away from the surge line with a sufficient surge margin, even though operation with a surge margin results in a performance penalty [2].

Much work has been conducted on surge control/management in various compression systems. Greitzer [3,4] developed an analytical model of gas turbine surge and investigated its physical mechanism. Based on Greitzer’s model [3,4], researchers have tried to control surge. Gyssling [5] conducted a linear stability analysis to investigate the effectiveness of plenum volume variation in stabilizing turbocharger surge. Willems et al. [6] focused on active surge control of a compression system in the presence of valve saturation and valve dynamics.

Analyzing instabilities in turbo heat pumps can be more complex than in gas turbines for the following reasons. First, a turbo heat pump is a closed-loop system, consisting of a compressor, condenser, throttle, and evaporator (see Fig. 1(a)). Thus, the compressor inlet is affected by its outlet. Second, there are two plenums, the condenser and evaporator, which are coupled to each other. Third, unlike air, the refrigerant (the working fluid in turbo heat pumps) exhibits strong real gas effects. Fourth, the turbo heat pump is a two-phase system: in the condenser and evaporator, the refrigerant exists in both vapor and liquid (the shaded region in Fig. 1(a)) phases. Lastly, the heat exchange between the refrigerant and the cooling water in the condenser and the chilled water in the evaporator has to be taken into account.

Recently, Kim and Song [7] developed and validated an analytical nonlinear model to predict hysteretic surge characteristics in turbo heat pumps. Despite such efforts, there has not yet been any effort to predict surge onset or to understand the corresponding physical mechanisms in turbo heat pumps. Therefore, this research presents physical, analytical, and experimental views of surge onset in turbo heat pumps. The main objectives are: (1) to investigate the physical mechanisms responsible for surge onset in turbo heat pumps, (2) to develop a new analytical stability model to predict surge onset in turbo heat pumps, (3) to validate the mechanisms and the model’s predictions with experiment, and (4) to understand how the system parameters influence surge onset.

Review of Surge Model

A typical turbo heat pump system is schematically shown in Fig. 1(a) and the corresponding thermodynamic cycle is shown in Fig. 1(b). The nonlinear model for surge characteristics in turbo heat pumps, recently developed by Kim and Song [7], forms the basis for the analysis of surge onset. Therefore, the model is briefly reviewed here and the details of the model are given in Appendix A. Kim and Song’s model consists of the following nine equations:

\[
d\hat{P}_1^1 = \frac{1}{B V_1} \left[ -\hat{m}_C^1 - (\hat{\rho}_1 \hat{\rho}_1 / \hat{\rho}_1^1) \hat{m}_T^1 + (\hat{\rho}_1^1 / \hat{\rho}_1^1 - 1) \hat{m}_a^1 \right] \tag{1}
\]

\[
d\hat{P}_2^2 = \frac{G}{B} \left( \frac{\rho_2}{\rho_2^2} / \hat{\rho}_2 \right) \left[ \hat{m}_C^2 + (\hat{\rho}_2 \hat{\rho}_2 / \hat{\rho}_2^2 - 1) \hat{m}_a^2 \right] \tag{2}
\]
The nondimensional input parameters which affect the system stability are

\[
B = \frac{U}{2\Omega_1 L_C}, \quad G = \frac{L_1 A_C}{L_C A_T}, \quad J = \frac{A^2_C}{A^2_T}
\]

\[
\frac{\omega_2}{\omega_1} = \frac{A_T}{A_C} \left( \sqrt[6]{\frac{V_{a1}^2}{V_{a2}^2}} \right)^{7/8}
\]

\[
\frac{Ma_1}{a_{1v,0}} = \frac{Ma_2}{a_{2v,0}} = \frac{U}{a_{1v,0}}
\]

\[
H_1 = \frac{\dot{m}_{ch_L} C_{pd} T_{ch_L}}{U^2}, \quad H_2 = \frac{\dot{m}_{ch_H} C_{pd} T_{ch_H}}{U^2}
\]

**Mass-Spring-Damper System Model and Physical Mechanisms**

**Mass-Spring-Damper Systems.** This section presents a new mass-spring-damper system of turbo heat pump systems based on the existing surge model [7]. The continuity equations for the condenser and evaporator (see Eqs. (1) and (2)) and the momentum equations for the compressor and evaporator (see Eqs. (5) and (6)) can be combined as follows:

\[
\frac{d^2 \dot{m}_C}{dt^2} - B \frac{d \dot{C}}{dt} \dot{m}_C + K_1 \left( \dot{m}_C - \dot{m}_{i1} \right) + \frac{\dot{p}_{1v}}{\rho_{1v}} \left( \dot{m}_{i1} - \dot{m}_T \right) = 0
\]

\[
- K_2 \left( \dot{m}_{c2} - \dot{m}_C \right) + \frac{\dot{p}_{2v}}{\rho_{2v}} \left( \dot{m}_T - \dot{m}_{c2} \right) = 0
\]

\[
G \frac{d^2 \dot{m}_T}{dt^2} + B \frac{d \dot{F}}{dt} \dot{m}_T - K_1 \left( \dot{m}_C - \dot{m}_{i1} \right) + \frac{\dot{p}_{1v}}{\rho_{1v}} \left( \dot{m}_{i1} - \dot{m}_T \right) + K_2 \left( \dot{m}_{c2} - \dot{m}_C \right) + \frac{\dot{p}_{2v}}{\rho_{2v}} \left( \dot{m}_T - \dot{m}_{c2} \right) = 0
\]

where \(K_1 = \frac{a_1^2}{V_1} \) and \(K_2 = G(\omega_2/\omega_1)^2(a_1^2/V_2)\).

Equations (10) and (11) are analogous to the equations of motion for a hypothetical coupled mass-spring-damper system with dampers and springs, as shown in Fig. 2. The masses move (without friction) only along the circular track shown. The effects of hypothetical dampers and springs are proportional to the tangential velocity and displacement, respectively, along the circular track. Even though \(d^2 \dot{m}/dt^2, \dot{d}m/dt, \) and \(d\) are actually the 3rd, 2nd, and 1st derivatives of mass with respect to time, these terms are analogous to and will be referred to as the acceleration, velocity, and displacement terms, respectively.

The acceleration terms \((d^2 \dot{m}/dt^2)\) correspond to the inertial force and \(G\) represents the ratio of the expansion valve duct inertia to the compressor duct inertia. The velocity terms \((d\dot{m}/dt)\) are the
damping forces and \(-Bd\ddot{C}/d\ddot{m}\) and \(Bd\ddot{F}/d\ddot{m}\) represent damping in the compressor and expansion valve. The displacement terms (\(n\)) are the restoring spring forces in the condenser and evaporator due to the compressibility of fluid. Finally, \(K_1\) and \(K_2\) are, respectively, the spring stiffnesses of the condenser and evaporator and they connect the inertias. The phase change rates \((\dot{m}_{1c} \text{ and } \dot{m}_{2c})\) in the displacement terms represent the rates of heat transfer in the condenser and evaporator. If the heat transfer effects in the condenser and evaporator are neglected by setting \(\dot{m}_{1c} = \dot{m}_{2c} = 0\) and \(\dot{\rho}_{1c}/\dot{\rho}_{2c} = \dot{\rho}_{2c}/\dot{\rho}_{3c} = 1\), the displacement terms in Eqs. (10) and (11) can be simplified to \(\pm (K_1 + K_2)(\dot{m}_{1c} - \dot{m}_{2v})\). Thus, the condenser and evaporator springs \(K_1\) and \(K_2\) are connected in parallel to the two inertias in the compressor and expansion valve ducts, indicating the closed loop structure of turbo heat pumps.

The condenser and evaporator stiffnesses \(K_1\) and \(K_2\) can be written as follows:

\[
K_1 = \frac{A_1^2}{K_{1v} A_1^2 + K_{1l} A_1^2} \\
K_2 = \frac{A_2^2}{K_{2v} A_2^2 + K_{2l} A_2^2} \\
\]

where the stiffness due to the liquid and vapor \((K_{1v}, K_{1l}, K_{2v},\) and \(K_{2l})\) are defined in Appendix B. In Eqs. (12) and (13) the stiffness due to the liquid and vapor in the heat exchangers are connected in series. Since liquid is essentially incompressible \(K_{1l} \gg K_{1v}\) and \(K_{2l} \gg K_{2v}\), respectively. Therefore, the liquid compliance effects \((1/K_{1l} + 1/K_{2l})\) are negligible relative to those of vapor.

The following additional assumptions are used to further simplify the multidegree-of-freedom system (see Fig. 2 and Eqs. (10) and (11)) to a single-degree-of-freedom system, as shown in Fig. 3:

1. quasi-steady compressor response
2. continuous disturbances in \(\dot{C}(\dot{m}_{1c}), \dot{m}_{1c}(\dot{m}_{1c}, \dot{m}_{2v})\), and \(\dot{m}_{2c}(\dot{m}_{1c}, \dot{m}_{2v})\)
3. negligible inertia in the compressor duct relative to the expansion valve duct inertia

Since the main focus of this study is surge onset, assumptions 1 and 2 are reasonable. The mass of liquid in the expansion valve duct is larger than that of the vapor in the compressor duct due to the large density difference between the liquid and vapor. Consequently, \(G\) which is the ratio of the expansion valve duct inertia to the compressor duct inertia, has been found to be 4.31 and 3.35 (from the experimental result of this paper). Therefore, assumption 3 is also reasonable, and, henceforth, the focus is on Eq. (11). Through the simplification process in Appendix C, the following single-degree-of-freedom system equation for the expansion valve inertia is obtained.

\[
G^2 \ddot{n}_{1c} + \left[ \frac{BF_0 - GS}{BC(M)} \right] \frac{d\dot{n}_{1c}}{dt} + \left[ R\dot{n}_{1c} - S \left( \frac{\dot{F}(\dot{m}_{1c}) - C(M)}{C(M)} \right) \dot{M} \right] = 0
\]

where

\[
S = K_1 \left( 1 + \frac{\dot{\rho}_{1c} - 1}{\dot{\rho}_{1l}} \frac{\partial \dot{n}_{1c}}{\partial \dot{m}_{1c}} \right) + K_2 \left( 1 + \frac{\dot{\rho}_{2c} - 1}{\dot{\rho}_{2l}} \frac{\partial \dot{n}_{2c}}{\partial \dot{m}_{2c}} \right) \left( \frac{\partial \dot{n}_{1c}}{\partial \dot{m}_{1c}} \right) \frac{\partial \dot{n}_{2c}}{\partial \dot{m}_{2c}} \\
R = K_1 \left( \frac{\dot{\rho}_{1c} - 1}{\dot{\rho}_{1l}} \frac{\partial \dot{n}_{1c}}{\partial \dot{m}_{1c}} \right) + K_2 \left( \frac{\dot{\rho}_{2c} - 1}{\dot{\rho}_{2l}} \frac{\partial \dot{n}_{2c}}{\partial \dot{m}_{2c}} \right) \\
W = K_1 \left( \frac{\dot{\rho}_{1c} - 1}{\dot{\rho}_{1l}} \frac{\partial \dot{n}_{1c}}{\partial \dot{m}_{1c}} \right) \left( 1 - \frac{\partial \dot{n}_{1c}}{\partial \dot{m}_{1c}} \right) \frac{\partial \dot{n}_{1c}}{\partial \dot{m}_{1c}} \\
\]

**Damping.** Surge is a self-sustained oscillation, or dynamic instability [1,3], which happens when the energy input into a system oscillation exceeds the energy dissipated by the system. In single-degree-of-freedom systems, positive damping leads to dynamically stable systems and negative damping results in dynamic instability. Therefore, zero damping can be thought of as the neutral stability point, or the onset of surge.

The system equation and damping in turbo heat pumps (see Eq. (14)) can be compared with that for gas turbines in Eq. (15) [3]

\[
\frac{d^2\dot{m}_{1c}}{dt^2} + \left[ \frac{1}{BF(M)} \right] \frac{d\dot{m}_{1c}}{dt} + \left[ \dot{m}_{1c} - \frac{F(M) - C(M)}{F'(M)} \right] = 0
\]

The system equation of gas turbines is derived for the compressor mass flow rate \((\dot{m}_{1c})\), whereas that of turbo heat pumps is done for the expansion valve mass flow rate \((\dot{m}_{2v})\) because the ratio of the expansion valve duct inertia to the compressor duct inertia \((G)\) is greater than 1 in turbo heat pumps but less than 1 in gas turbines \((G = 0.36\) in Greitzer’s study [3]). As in gas turbines, damping in turbo heat pumps is provided by not only the compressor but also other system components. The compressor characteristic slope \((C)\), expansion valve characteristic slope \((F')\), and \(B\) are important in the damping of both systems. Additionally, \(G\) and \(S\) also affect damping in turbo heat pumps. The \(S\) factor in Eq. (14) represents the influence of the heat exchange, closed loop structure, two plenums, and the two phase working fluid. The heat transfer effect is found in the rates of phase change in the condenser and evaporator per unit compressor mass flow rate \(\partial \dot{m}_{1c}/\partial \dot{m}_{1c}\) \(\dot{M}\) and \(\partial \dot{m}_{2c}/\partial \dot{m}_{2c}\) \(\dot{M}\). The closed loop two plenum structure is shown in the springs \((K_1 \text{ and } K_2)\). Finally, the two phase flow is reflected in the density ratios \((\dot{\rho}_{1c}/\dot{\rho}_{1l} \text{ and } \dot{\rho}_{2c}/\dot{\rho}_{2l})\) and the composition of the springs \((K_1 \text{ and } K_2)\).

**Surge Onset.** In Eq. (14), \(S, G, B, \text{ and } \dot{F}\) are always positive, but the slope of the centrifugal compressor characteristic \((C'(M))\) changes sign from negative to positive as the flow coefficient is decreased. At high flow rates, \((C'(M))\) is negative, system damping
is positive, and the turbo heat pump systems are stable. At low flow
rates, \( C'(M) \) can be positive, resulting in negative system damping,
or turbo heat pump instability (e.g., surge). Thus, the sign change in
\( C'(M) \) is a necessary condition for turbo heat pump surge onset.

Usually, reciprocating, screw, and centrifugal compressors are
used in small, medium, and large capacity heat pumps, respectively.
Among these, only heat pumps with centrifugal compressors suffer
from surge at off design conditions. Unlike centrifugal compressors,
reciprocating and screw compressors always have negative character-
istic slopes [8] and, thus, Eq. (14) explains why heat pumps with
reciprocating or screw compressors do not suffer from surge.

According to Eq. (14), the system damping of turbo heat pumps
becomes zero when the compressor characteristic slope satisfies
Eq. (16)

\[
\hat{C}'(\hat{M}) = \frac{GS}{B^2}F
\]

(16)

The surge onset prediction from Eq. (16) is subsequently verified
against experimental data and is used to explain parametric trends
in the validation section.

**Linearized Stability Model**

This section presents the new stability analysis based on the
linearization of the nonlinear surge model (see Eqs. (1)–(9)). In the
linearized model, all of the flow variables are assumed to be composed
of mean (\( \bar{\cdot} \)) and perturbation (\( \delta(\cdot) \)) parts. For example, the mass
flow rates in the compressor and throttle can be expressed as

\[
\hat{m}_C = \bar{m}_C + \delta\hat{m}_C
\]

(17)

\[
\hat{m}_T = \bar{m}_T + \delta\hat{m}_T
\]

(18)

and the compressor pressure rise and expansion valve pressure drop as

\[
\hat{C} = \bar{C} + \left( \frac{dC}{dm} \right) \delta\hat{m}_C
\]

(19)

\[
\hat{F} = \bar{F} + \left( \frac{dF}{dm} \right) \delta\hat{m}_T
\]

(20)

Likewise, the compressor outlet and inlet pressures, and the rate
of phase change in the heat exchangers, can be expressed as

\[
\hat{P}_1 = \bar{P}_1 + \delta\hat{P}_1
\]

(21)

\[
\hat{P}_2 = \bar{P}_2 + \delta\hat{P}_2
\]

(22)

\[
\hat{\dot{m}}_{s1} = \bar{\dot{m}}_{s1} + \left( \frac{d\dot{m}_{s1}}{dP} \right) \delta\hat{P}_1
\]

(23)

\[
\hat{\dot{m}}_{s2} = \bar{\dot{m}}_{s2} + \left( \frac{d\dot{m}_{s2}}{dP} \right) \delta\hat{P}_2
\]

(24)

After eliminating the mean components, the nondimensional
perturbation equations of the system can be obtained as

\[
\frac{d\delta\hat{m}_C}{dt} = B \left[ -\delta\hat{P}_1 + \delta\hat{P}_2 + \left( \frac{d\bar{C}}{dm} \right) \delta\hat{m}_C \right]
\]

(25)

\[
\frac{d\delta\hat{m}_T}{dt} = B \left[ \delta\hat{P}_1 - \delta\hat{P}_2 - \left( \frac{d\bar{F}}{dm} \right) \delta\hat{m}_T \right]
\]

(26)

\[
\frac{d\delta\hat{P}_1}{dt} = B \frac{1}{\bar{V}_1} \left[ \delta\hat{m}_C - \left( \frac{\bar{\dot{p}}_{t1}}{\bar{\dot{p}}_{t1}} \right) \delta\hat{m}_T + \left( \frac{\bar{\dot{p}}_{s1}}{\bar{\dot{p}}_{s1}} - 1 \right) \left( \frac{d\hat{m}_{s1}}{dP} \right) \delta\hat{P}_1 \right]
\]

(27)

\[
\frac{d\delta\hat{P}_2}{dt} = G \frac{1}{\bar{V}_2} \left[ \delta\hat{m}_C + \left( \frac{\bar{\dot{p}}_{s2}}{\bar{\dot{p}}_{s2}} \right) \delta\hat{m}_T + \left( \frac{\bar{\dot{p}}_{2s}}{\bar{\dot{p}}_{2s}} - 1 \right) \left( \frac{d\hat{m}_{s2}}{dP} \right) \delta\hat{P}_2 \right]
\]

(28)

Equations (25)–(28) can then be rewritten in matrix form as
shown in Eq. (29). Assuming that each perturbation quantity has
the form \( e^{\lambda t} \), the linearized turbo heat pump stability model can be
treated as an eigenvalue problem, where \( \lambda \) is the eigenvalue of
the stability matrix \( M \) [9]. When the real part of the eigenvalue
becomes positive, the system becomes unstable.

\[
\dot{Y} = MY = \begin{bmatrix}
\frac{d\delta m_C}{dt} \\
\frac{d\delta P_1}{dt} \\
\frac{d\delta m_T}{dt} \\
\frac{d\delta P_2}{dt}
\end{bmatrix} = \begin{bmatrix}
B \left( \frac{d\bar{C}}{dm} \right) & -B & 0 & B \\
\frac{1}{\bar{V}_1} \left( \frac{\bar{\dot{p}}_{t1}}{\bar{\dot{p}}_{t1}} \right) - 1 & \frac{1}{\bar{V}_1} \left( \frac{\bar{\dot{p}}_{s1}}{\bar{\dot{p}}_{s1}} - 1 \right) \left( \frac{d\bar{m}_{s1}}{dP} \right) & 0 & 0 \\
0 & B & -B & -\frac{B}{G} \\
-G \frac{1}{\bar{V}_2} \left( \frac{\bar{\dot{p}}_{s2}}{\bar{\dot{p}}_{s2}} \right) & 0 & 0 & \frac{G}{\bar{V}_2} \left( \frac{\bar{\dot{p}}_{2s}}{\bar{\dot{p}}_{2s}} - 1 \right) \left( \frac{d\bar{m}_{s2}}{dP} \right)
\end{bmatrix}
\]

(29)
Validation of Predictions

**Turbochargers.** If the following assumptions are applied to Eq. (29); the stability matrix \( M \) becomes a 2 by 2 matrix in terms of \( \delta \dot{P}_1 \) and \( \delta \dot{m}_C \).

- no heat exchange in the heat exchangers: \( \dot{m}_1 = \dot{m}_2 = 0 \)
- fixed compressor inlet pressure: \( \delta \dot{P}_2 = 0 \)
- single phase condition: \( V_{1i} = V_{2i} = 0 \)
- negligible inertia in the expansion duct: \( \delta \dot{m}_T / dt = 0 \)

The simplified matrix in Eq. (30) is then identical to Greitzer’s model [3].

\[
\begin{bmatrix}
\frac{d\delta \dot{m}_C}{dt} \\
\frac{d\delta \dot{P}_1}{dt}
\end{bmatrix} =
\begin{bmatrix}
B \frac{dC}{dm} & -B \\
\frac{1}{B} & \frac{1}{B} \frac{dF}{dm}
\end{bmatrix}
\begin{bmatrix}
\delta \dot{m}_C \\
\delta \dot{P}_1
\end{bmatrix}
\]  

Equation (30) has been applied to predict surge onset in a turbocharger centrifugal compressor studied by Pinsley [10] and Gysling [5]. The compressor characteristic of Pinsley [10] (also used by Gysling [5]) has been used as input (see Fig. 4). In the figure, the experimentally determined surge onset point at \( \phi = 0.116 \) is also indicated. For the calculation, the value of the \( B \) parameter is taken to be 1.0 and the compressor characteristic is represented with a 3rd order polynomial. The root locus plots from both the Greitzer model and the new turbo heat pump stability model are shown in Fig. 5. The roots are plotted for decreasing flow coefficients ranging from 0.175 to 0.070. As the flow coefficient decreases, the eigenvalues move from the left half plane (stable) to the right half plane (unstable). The neutral stability point on the imaginary axis corresponds to the flow coefficient value of 0.116. The new model’s predictions agree well with those from Greitzer’s model and experimental data. Thus, the new model can accurately predict surge onset in single-phase open-loop systems such as turbochargers.

**Turbo Heat Pumps.** This section compares predictions from the mass-spring-damper system and the new stability model to the experimental data. Experiments have been conducted in the turbo heat pump test facility at LG Electronics (see Fig. 6).

**Experimental Facility.** The main flow loop consists of a centrifugal compressor with a vaned diffuser, condenser, orifice plate as the expansion valve, and evaporator. A 400-kW DC motor...
powers, via a gearbox, a 400-RT1 compressor, which circulates refrigerant R134a through the loop. The condenser and evaporator are of the shell-and-tube type, consisting of a large pressure vessel (shell) with a bundle of tubes inside. Cooling water runs through the tubes in the condenser and removes heat from the vapor refrigerant. Chilled water runs through the tubes in the evaporator and transfers heat to the liquid refrigerant.

**Instrumentation.** In Fig. 7, the types and locations of the sensors are indicated. Here, \( P \) and \( T \) refer to the pressure and temperature sensors and \( F \) to the flow meter. Resistance temperature detectors are positioned at the inlets and outlets of the compressor, cooling water tubes, and chilled water tubes. Siemens 7MF4332 pressure transducers are used to measure the pressure at the inlets and outlets of the compressor and the two heat exchangers. The same type of pressure transducers are used to measure the vapor refrigerant pressure in the heat exchangers. The mass flow rates of the refrigerant and the cooling/chilled water are measured by two types of sensors. Krohne OPTIFLUS 4300 C flowmeters are used at the cooling water inlet and the chilled water inlet. A Sierra 240 vortex flowmeter measures the flow velocity at the compressor exit. The vortex flowmeter can measure the magnitude but cannot detect the direction of the mass flow. Therefore, flow reversal has been determined by comparing the compressor inlet and exit temperatures as in Fink et al. [12]. Other properties such as the density, speed of sound, quality, and saturation temperature are calculated as functions of the pressure using REFPROP [13].

**Test Compressors.** Two different compressors have been tested. The specific compressor and system parameters are listed in Table 1. At the design condition of compressor A, the flow coefficient, work coefficient, and specific speed are 1.27, 0.272, and 0.130, respectively. For compressor B, the corresponding values are, respectively, 2.10, 0.230, and 0.125.

### Table 1 Compressor and system parameters

<table>
<thead>
<tr>
<th>Compressor</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow coefficient</td>
<td>0.272</td>
<td>0.230</td>
</tr>
<tr>
<td>Work coefficient</td>
<td>1.27</td>
<td>2.10</td>
</tr>
<tr>
<td>Specific speed (Ns)</td>
<td>0.130</td>
<td>0.125</td>
</tr>
<tr>
<td>( B )</td>
<td>1.72</td>
<td>2.52</td>
</tr>
<tr>
<td>( G )</td>
<td>4.31</td>
<td>4.35</td>
</tr>
<tr>
<td>( J )</td>
<td>576</td>
<td>391</td>
</tr>
<tr>
<td>( \omega_2/\omega_1 )</td>
<td>0.461</td>
<td>0.477</td>
</tr>
</tbody>
</table>

Comparison of Predictions and Experimental Data: Compressor A. Figure 8 shows the measured compressor characteristic with the measured surge onset at \( \phi = 0.178 \). For analysis, the compressor characteristic for compressor A has been represented with a 4th order polynomial (the solid line in Fig. 8). According to Eq. (16) of the mass-spring-damper system, surge onset in compressor A is predicted to occur at \( \phi = 0.170 \) and the prediction agrees well with the measured surge onset flow coefficient. Figure 9 shows the root locus predictions from the linear turbo heat pump stability model (see Eq. (29)) plotted as a function of the flow coefficient. As the flow coefficient is reduced from 0.178 to 0.173, the poles gradually move to the right-hand plane from the left-hand plane. Thus, the linear stability model predicts surge onset at \( \phi = 0.175 \) for compressor A and this prediction also matches well with the measured surge onset flow coefficient.

Comparison of Predictions and Experimental Data: Compressor B. Figure 10 shows the measured compressor characteristic with the measured surge onset at \( \phi = 0.193 \). The compressor characteristic for compressor B has been represented with a 3rd order polynomial (the solid line in Fig. 10). The mass-spring-damper system (see Eq. (16)) predicts surge onset at \( \phi = 0.194 \) and the prediction matches well with the experimental observation. Figure 11 shows the root locus predictions from the linear turbo heat pump stability model (see Eq. (29)). As the flow coefficient is

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\[ F = 3.024 \text{ kcal/h} \]
Parametric Study

From the mass-spring-damper system, system damping of the turbo heat pumps includes $C^1$, $F^1$, $S$, $G$, and $B$. Among them, $S$ is a function of two independent variables: $a_{2}/a_{1}$ and $G$. Thus, system damping is determined by $C^1$, $F^1$, $a_{2}/a_{1}$, $G$, and $B$. Furthermore, compressor and expansion valve selections fix $C^2$ and $F^2$. Therefore, in this section, the parametric influence of $a_{2}/a_{1}$, $G$, and $B$ on surge onset is investigated. For parametric predictions, the linear stability model has been used with the characteristic of compressor B as the baseline case. Except for the parameter of interest, the values of the other parameters have been kept constant at the baseline values. The parametric influence of the system parameters is physically explained by using the mass-spring-damper system.

Effect of $a_{2}/a_{1}$. Here, $a_{2}/a_{1}$ refers to the ratio of the Helmholtz resonator frequencies of the condenser and evaporator and is a function of the speed of sound ($a_{1}$ and $a_{2}$), $G$, and the condenser and the evaporator volumes ($V_{1}$ and $V_{2}$) (see Eq. (31)). Therefore, keeping $G$ (a function of $A_{C}$, $A_{T}$, $L_{C}$, and $L_{T}$) and $B$ (a function of $V_{1}$, $A_{C}$, and $L_{C}$) constant, $a_{2}/a_{1}$ can be independently increased by decreasing the evaporator volume ($V_{2}$).

$$
\frac{a_{2}}{a_{1}} = \left(\frac{d_{20,0}}{d_{10,0}}\right)^{1/3} \frac{V_{1,0}}{\sqrt{V_{2,0}}} \approx \frac{1}{\sqrt{G}} \frac{K_{2,0}}{K_{1,0}}
$$
(31)

Figure 12 shows the surge onset prediction with varying $a_{2}/a_{1}$ on the compressor characteristic. For varying $a_{2}/a_{1}$ values of 0.477, 0.955, 1.43, 1.91, and 2.39, surge onset is predicted at 0.193, 0.190, 0.188, 0.173, and 0.168, respectively. Thus, as $a_{2}/a_{1}$ increases, surge onset is predicted at lower flow coefficients. This tendency is qualitatively consistent with the predictions of the mass-spring-damper system (see Eq. (16)). An increase in $a_{2}/a_{1}$ increases $S$ and the right-hand-side in Eq. (16) and results in surge onset at a lower flow coefficient. Physically, $a_{2}/a_{1}$ can be explained in terms of the inertia ratio ($G$) and the stiffness ratio ($K_{2,0}/K_{1,0}$) as written in Eq. (31). Thus, an increase in $a_{2}/a_{1}$ with constants $G$ and $B$ means an increase in the evaporator stiffness ($K_{2,0}$), or increased system resistance to a given external disturbance, resulting in surge onset at a lower flow coefficient.

Effect of $G$. Here, $G$ is geometric parameter of the compressor and expansion valve ducts (see the Nomenclature section). An independent increase of $G$ with constants $J$, $a_{2}/a_{1}$, and $B$ can be obtained by an increase in the expansion valve duct length (see Eq. (32))

$$
G = \frac{L_{T}/A_{T}}{L_{C}/A_{C}} = \frac{L_{T}}{L_{C}} \sqrt{J}
$$
(32)

Figure 13 shows model predictions for different $G$ values. With $G = 4.35, 8.69, 13.0, 17.4$, and $21.7$, the stability model predicts surge onset at $\phi = 0.193, 0.192, 0.191, 0.190$, and $0.188$, respectively. Thus, increasing $G$ results in surge onset at lower flow coefficient values. This trend is, again, qualitatively consistent.
with that predicted by the mass-spring-damper system (see Eq. (16)). An increase in \( G \) increases the right-hand-side in Eq. (16) and results in surge onset at a lower flow coefficient. Physically, an increase in \( G \) for constants \( J, \omega_2/\omega_1 \), and \( B \) means increases in the expansion valve duct inertia and evaporator stiffness. Thus, the stability of the system is enhanced, resulting in surge onset at a lower flow coefficient.

**Effect of \( B \).** The \( B \) parameter is a well-known important factor in determining the dynamics of compression systems. Assuming a constant impeller tip speed \( U \) (see Eq. (33)), \( B \) is proportional to the square root of the ratio of the condenser volume \( (V_{1,0}) \) and the compressor duct area \( (A_C) \) and length \( (L_C) \). With constants \( \omega_2/\omega_1 \) and \( G, B \) can be varied by changing \( V_{1,0}, A_C, \) and \( L_C \) while maintaining the ratios \( V_{1,0}/V_{2,0}, A_C/A_T, \) and \( L_C/L_T \) constant.

\[
B = \frac{U}{2\omega_1L_C} = \frac{U}{2\omega_1V_{1,0}} \sqrt{\frac{V_{1,0}}{A_C L_C}} \propto \frac{1}{\sqrt{\text{condenser stiffness} \times \text{compressor duct inertia}}} \quad (33)
\]

Figure 14 shows the predicted surge onset points for varying \( B \) parameters. When the \( B \) parameter has the value of 2.52, surge onset occurs at \( \phi = 0.193 \). With \( B = 1.51, 1.26, 1.01, \) and 0.757, the turbo heat pump system becomes unstable at \( \phi = 0.191, 0.188, 0.185, \) and 0.176, respectively. Thus, a rise in \( B \) results in surge onset at higher flow coefficients. This trend is consistent with the predictions of the mass-spring-damper system (see Eq. (16)). According to Eq. (16), contrary to \( \omega_2/\omega_1 \) and \( G \), a decrease in \( B \) results in surge onset at a lower flow coefficient. Here, \( B \) can be thought of as a function of the condenser stiffness and compressor duct inertia (see Eq. (33)). A decrease in \( B \) with constants \( \omega_2/\omega_1 \) and \( G \) means increases in all of the compressor duct inertia, condenser stiffness, expansion valve duct inertia, and evaporator stiffness, enabling the flow to better withstand given flow perturbations.

**Conclusions**

The conclusions from this physical, analytical, and experimental investigation of surge onset in turbo heat pumps can be summarized as follows:

1. Physical insight into surge onset mechanisms in turbo heat pumps has been obtained via a mass-spring-damper system based on the simplification of an existing turbo heat pump surge model.
2. Turbo heat pumps can be considered as a mass-spring-damper system with two inertias (compressor and expansion valve ducts), two dampers (compressor and expansion valve), and four springs (vapor and liquid phases in the condenser and evaporator). Two dampers are separately connected to the inertias. Condenser and evaporator springs are each composed of vapor and liquid springs connected in series. Finally, the two inertias are connected by condenser and evaporator springs in parallel.
3. The mass-spring-damper system can be further simplified into a single degree-of-freedom system in which surge onset occurs at the neutral stability point with zero system damping. System damping is determined by the slope of the compressor characteristic \((C')\), the slope of the expansion valve characteristic \((F')\), Greitzer's system parameter \((B)\), the inertia ratio of the compressor and expansion valve ducts \((G)\), and the combined effects of heat transfer, two plenums, closed loop structure, and two-phase flow \((S)\).
4. The sign change in the slope of the compressor characteristic is a necessary condition for surge onset and this necessity explains why turbo heat pump surge takes place in heat pumps with centrifugal compressors but not in systems with reciprocating or screw compressors.
5. A linear stability model for turbo heat pumps, based on the first principles, has been developed and validated.
6. Comparison with experimental results shows that the mass-spring-damper system model and the linearized stability model can both accurately predict surge onset in turbo heat pumps.
7. An increase in \( \omega_2/\omega_1 \) with constants \( G \) and \( B \) represents increased evaporator stiffness, or increased system resistance to external disturbance. Thus, surge onset takes place at a lower flow coefficient.
8. An increase in \( G \) for constants \( \omega_2/\omega_1 \) and \( B \) means increases in the expansion valve duct inertia and evaporator stiffness, resulting in surge onset at a lower flow coefficient.
9. A decrease in \( B \) for constants \( \omega_2/\omega_1 \) and \( G \) means increases in all of the compressor inertia, condenser stiffness, expansion valve duct inertia, and evaporator stiffness. Thus, the system is stabilized with decreasing \( B \).

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**Greek Symbols**

- $\lambda$ = eigenvalue
- $\rho$ = density, kg/m$^3$
- $\Phi$ = flow coefficient
- $\Psi$ = work coefficient
- $\omega$ = Helmholtz resonator frequency, $a\sqrt{A/\nu}$, 1/s

**Subscripts**

- $C$ = compressor
- chw = chilled water
- cw = cooling water
- $l$ = liquid refrigerant
- $s$ = phase change
- SS = steady state
- $T$ = expansion valve
- $v$ = vapor refrigerant
- $x$ = two-phase properties
- 0 = initial value or inlet value
- 1, $H$ = condenser
- 2, $L$, evaporator

**Operators**

- $(\cdot)'$ = nondimensional variable
- $(\cdot)_{av}$ = time averaged value
- $(\cdot)^{\delta}$ = perturbation quantity

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**Appendix A: Nonlinear Surge Model**

In a typical turbo heat pump system (see Fig. 1) the refrigerant vapor is compressed in the compressor and enters the condenser. In the condenser, the refrigerant vapor releases heat to the cooling water and is condensed into the liquid phase. The liquefied refrigerant is then throttled via the expansion valve and enters the evaporator. The liquid refrigerant then is evaporated by absorbing heat from the chilled water in the evaporator and the cycle repeats. To model surge in turbo heat pumps, the following assumptions are used:

1. The flow is inviscid and one-dimensional.
2. The refrigerant is R134a.
3. The flow is spatially uniform for both liquid and vapor phases.
4. Across the heat exchangers and ducting, the pressure drop is negligible.
5. Gravitational effects are negligible.
6. The mass flow rates and inlet temperatures of the cooling water and chilled water are constant.
7. The condenser and evaporator are shell-and-tube type heat exchangers with the refrigerant flowing inside the shell and cooling and chilled water flowing inside the tubes.
8. At the condenser inlet, the temperature of the cooling water is lower than the saturation temperature of the refrigerant in the condenser. At the condenser exit, the cooling water’s temperature is equivalent to the refrigerant’s saturation temperature.
9. At the evaporator inlet, the chilled water temperature is higher than the saturation temperature of the refrigerant in the evaporator. At the evaporator exit, the chilled water’s temperature is equal to the saturation temperature of the refrigerant.
10. The refrigerant at the condenser inlet and evaporator exit is a saturated vapor. At the condenser exit, the refrigerant is a saturated liquid. At the exit of the expansion valve, the refrigerant is a mixture of the vapor and liquid phases. At the compressor exit, the refrigerant is slightly superheated; however, the amount of superheat is small and, thus, negligible.

With such assumptions, the turbo heat pump model consists of nine equations: four mass conservation equations for the liquid and vapor phases in the condenser and evaporator (see Eqs. (A1)–(A4)), two momentum conservation equations across the compressor and expansion valve (see Eqs. (A5) and (A6)), and two energy conservation equations for the condenser and evaporator (see Eqs. (A7) and (A8)), and one compressor transient response equation (see Eq. (A9))
With the following nondimensionalization, nondimensional governing equations (see Eqs. (1)–(9)) and nondimensional parameters can be obtained

\[ \tilde{m} = \frac{m}{\rho_2 V_0}, \quad \tilde{P} = \frac{P}{\rho_2 V_0 U^2}, \quad \tilde{t} = \omega_1 t \]

\[ \tilde{V}_{1(1)} = \frac{V_{1(1)}}{V_{1,0}}, \quad \tilde{V}_{2(2)} = \frac{V_{2(2)}}{V_{2,0}} \]

\[ \tilde{p} = \frac{p}{\rho_2 V_0}, \quad \tilde{h}_{fg} = \frac{h_{fg}}{U} \]

\[ \tilde{a}_{1(1)} = \frac{a_{1(1)}}{a_{1,0}}, \quad \tilde{a}_{2(1,2)} = \frac{a_{2(1,2)}}{a_{2,0}} \]

\[ \tilde{T}_1 = \frac{T_1}{T_{ch,0}}, \quad \tilde{T}_2 = \frac{T_2}{T_{ch,0}} \]

**Appendix B: Definition of Helmholtz Resonator Frequency and Stiffnesses**

The Helmholtz resonator frequencies of the liquid and vapor in the condenser and evaporator can be defined as follows:

\[ \omega_{1v} = a_{1v} \frac{A_c}{V_{1,v} L_c} \]  
\[ \omega_{1l} = a_{1l} \frac{A_f}{V_{1,l} L_f} \]  
\[ \omega_{2v} = a_{2v} \frac{A_c}{V_{2,v} L_c} \]  
\[ \omega_{2l} = a_{2l} \frac{A_f}{V_{2,l} L_f} \]

Next, using the mass-stiffness-frequency relation, the dimensional spring stiffness of the liquid and vapor in the condenser and evaporator are defined as follows:

\[ k_{1v} = a_{1v} \frac{\rho_1 A_c L_c}{V_{1,v} L_c} \]  
\[ k_{1l} = a_{1l} \frac{\rho_1 A_f L_f}{V_{1,l} L_f} \]  
\[ k_{2v} = a_{2v} \frac{\rho_2 A_c L_c}{V_{2,v} L_c} \]  
\[ k_{2l} = a_{2l} \frac{\rho_2 A_f L_f}{V_{2,l} L_f} \]  
\[ k_{1v,0} = a_{1v} \frac{\rho_1 A_c L_c}{V_{1,v} L_c} \]

The nondimensional spring stiffnesses of the liquid and vapor in the condenser and evaporator are defined as follows:

\[ K_{1v} = k_{1v,0} \]  
\[ K_{1l} = k_{1l,0} \]  
\[ K_{2v} = k_{2v,0} \]  
\[ K_{2l} = k_{2l,0} \]

**Appendix C: Simplification Into a Single Degree-of-Freedom Equation**

The multi degree-of-freedom equation (see Eq. (11)) can be simplified into a single degree-of-freedom equation (see Eq. (14)) as follows. To describe \( \tilde{m}_C \) in terms of \( \tilde{m}_Y \), the compressor pressure rise \( \tilde{C} \) is linearized about a mean flow rate \( M \) as follows:

\[ \tilde{C}(\tilde{m}_C) = \tilde{C}(\tilde{M}) + \tilde{C}'(\tilde{M})(\tilde{m}_C - \tilde{M}) \]  
\[ \text{where } \tilde{C}' = \frac{d\tilde{C}}{d\tilde{m}_C} \cdot \]

By neglecting the inertia in the compressor duct (see Eq. (5)), the compressor pressure rise is:

\[ \tilde{C}(\tilde{m}_C) = \tilde{F}(\tilde{m}_Y) + \frac{G}{B} \frac{d\tilde{m}_Y}{dt} \]

Finally, by combining Eqs. (C1) and (C2), \( \tilde{m}_C \) can be expressed as a function of \( \tilde{m}_Y \) as follows:

\[ \tilde{m}_C = \frac{[\frac{G}{B} \frac{d\tilde{m}_Y}{dt} + \tilde{F}(\tilde{m}_Y) - \tilde{C}(\tilde{M})]}{\tilde{C}'(\tilde{M})} + \tilde{M} \]

Finally, by substituting Eqs. (C3)–(C5) into Eq. (11), Eq. (14) is obtained.

**References**